Voter models of social opinion formation

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Scopus – document search
(title, abstract, key words)
Agenda

• How to describe an opinion?
• What is opinion dynamics?
  – Linear voter model
  – Sznajd model
  – Q-voter model
• What are the interesting questions?
  – Theoretical studies
  – Interdisciplinary applications
Opinion dynamics

Social mood:
Is the situation in our country getting better or worse?

CBOS
How to define “opinion”?

- **Opinion** - verbalized attitude (Trommsdorff 1998), measured in surveys

- **Attitude** - is a hypothetical construct that represents an individual's like or dislike for an item
One of the most common in empirical social influence experiments
(Robins, *Handbook of research methods in personality psychology*, 2007)

If measured on a multi-level scale
- the distribution of opinions on “important” issues is typically bimodal (peaks at extreme values)
- mapping onto a dichotomous format
Voter models – binary opinions

SPINSON = SPIN + PERSON

- Basic dynamical rule
  - Infection
  - Imitation
  - Conformity

Public opinion

\[ m = \frac{1}{N} (N_+ - N_-) = \frac{1}{N} \sum S_i \]
Linear voter model
Clifford & Sudbury (1973), Holley & Liggett (1975)

• Each node can be in one of two discrete states
• VM evolution:
  – Pick a random node - a voter
  – The voter adopts the state of a random neighbor
One is not enough!

- Milgram, Bickman & Berkowitz (1969)
- Results of experiments: 1 → 4%, 4-5 → 80%
- Robert B. Cialdini:
  - Social validation, the fundamental way of decision making
Conformity

• Conformity – the main mechanism of collective actions
  – Informational: “when in doubt, imitate”
  – Normative: “when in Rome, do as the Romans do”
  Aversion to standing out in the crowd

• Conformity is relative!
Asch’s experiment

- Normative influence
- Asch (1956) „visual perception”
The size of the group is important...

![Graph showing conformity vs. size of the group](image)

- **Asch (1956)**
- **Milgram et al (1969)**
Even more surprising ...
Unanimity is the key!

• The presence of a social supporter reduced the total number of yielding responses from 32% to 5.5%!
• Participants were far more independent when they were opposed by
  – a 7-person majority and had a partner
  – than when they were opposed by a 3-person majority and did not have a partner
Conformity and the group size

• The threshold value of the group size varies between 3 and 5

• Review on the theory and experiments (Bond, 2005)
  – The size of the group cannot be too small, i.e., has to be of a sufficient size to invoke the social pressure
  – The size of the group cannot be too large

• All experiments with unanimous majority
Sznajd model
Sznajd-Weron & Sznajd, 2000, IJMPC

• $S_i = \pm 1$
• If $S_i S_{i+1} = 1$ then $S_{i-1} = S_{i+2} = S_i$
Sznajd model in 2D (Stauffer, 2000)

Nonlinear $q$-voter model
Castellano, Muñoz & Pastor-Satorras, 2009, PRE

- Each individual interacts with a set of $q$ of his nearest neighbors
- If all $q$ neighbors share the same state, the individual conforms to this state
- Otherwise, the individual flips with probability $\epsilon$
How to model conformity?

- q-voter
- Ising at $T=0$
- Threshold model ($T=1$)
- Majority (Galam, Redner)

Differences between models and ... 

- SM can be completely reformulated in terms of a linear VM (Behera & Schweitzer, 2003)

United we stand, divided we fall (Sznajd-Weron & Sznajd, 2000)

If you don’t know what to do, do nothing
Exit probability for the Voter and Sznajd models

Probability to reach an absorbing state with all spinsons + (up), provided that at the beginning the fraction of + was $x$?

$$E(x) = \frac{x^2}{2x^2 - 2x + 1}$$

$$= \frac{x^2}{x^2 + (1 - x)^2}$$

FIG. 4: Exit probability $E(x)$ for the Sznajd model. The system is one dimensional and composed of 25 and 100 voters respectively. Results are averaged over 5000 realizations of the random process.

- Lambiotte et al., *EPL* 82 18007 (2008)
- Slanina et al., *EPL* 82 18006 (2008)
Exit probability for the q-voter model

An analytical expression for the exit probability of the q-voter model in one dimension

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(Dated: August 13, 2014)

We present in this paper an approximation that is able to give an analytical expression for the exit probability of the q-voter model in one dimension. This expression gives a better fit for the more recent data about simulations in large networks [7], and as such, departs from the expression $\frac{\rho^q}{\rho^q+(1-\rho)^q}$ found in papers that investigated small networks only [4–6]. The approximation consists in assuming a large separation on the time scales at which active groups of agents convince inactive ones and the time taken in the competition between active groups. Some interesting findings are that for $q = 2$ we still have $\frac{\rho^2}{\rho^2+(1-\rho)^2}$ as the exit probability and for large values of $q$ the difference between the result and $\frac{\rho^q}{\rho^q+(1-\rho)^q}$ becomes negligible (the difference is maximum for $q = 5$ and 6)

$$E(x) = \frac{x^q}{x^q + (1 - x)^q}$$
Sznajd model in 2D (Stauffer, 2000)

Voter vs. Invasion (Rumor) process

• VM evolution (inflow):
  – Pick a random node - a voter
  – The voter adopts the state of a random neighbor

• IP evolution (outflow):
  – Pick a random node – an invader
  – The invader exports its state to a random neighbor

Surface tension – initial disordered state

Voter model

Glauber for $T=0$

Sznajd model
The dynamics of a droplet?

- Voter model
- Glauber for $T=0$
- Sznajd model
Is the outflow equivalent with the inflow?


- One dimension, $q=2$
- Heterogeneous lattice, $q=1$
- One dimension, $q>2$?
Some details are important, other not

• Inflow vs. outflow
• Sequential or synchronous updating?
• Anticonformity or independence?
• Person or situation?
Synchronous or sequential update

- Sequential:

- Synchronous:

A story of Hush Puppies

- The Hush Puppies brand was founded in 1958
- The classic American brushed-suede shoes with the lightweight crepe sole
- 1994 – sales of Hush Puppies were down to 30,000 pairs a year
- And suddenly ...
A story of Hush Puppies

- Hush Puppies had suddenly become hip in the clubs and bars of downtown Manhattan
- By the fall of 1995 several designers wanted to use them in their Spring collections
- In 1995, the company sold 450,000 pairs of the classic Hush Puppies
A story of Hush Puppies

• How did this happen?
• The first few kids were not deliberately trying to promote Hush Puppies
• They were wearing them because no one else would wear them
• No one was trying to make Hush Puppies a trend
• Initially – anticonformity
The diamond model: social response
Nail et al. (2000)

<table>
<thead>
<tr>
<th>Preexposure</th>
<th>Postexposure</th>
</tr>
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<tbody>
<tr>
<td>Agreement</td>
<td>Disagreement</td>
</tr>
<tr>
<td>Disagreement</td>
<td></td>
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</tbody>
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- **Conformity (c)**: When there is pre-exposure to disagreement and post-exposure to agreement, the response is movement towards conformity. The symbols indicate that three individuals move from disagreement to agreement.

- **Independence (i)**: When there is pre-exposure to agreement and post-exposure to disagreement, the response is movement towards independence. The symbols indicate that three individuals move from agreement to disagreement.

- **Uniformity (u)**: When there is pre-exposure to agreement and post-exposure to agreement, the response is movement towards uniformity. The symbols indicate that three individuals move from agreement to agreement.

- **Anticonformity (a)**: When there is pre-exposure to disagreement and post-exposure to disagreement, the response is movement towards anticonformity. The symbols indicate that three individuals move from disagreement to disagreement.

Q-voter model with non-conformity

\[ p \rightarrow \text{Non-conformity} \]
\[ (1 - p) \rightarrow \text{conformity} \]

\[ \epsilon = 0 \]


If you don’t know what to do, do whatever (Sznajd-Weron & Sznajd, 2000)
Q-voter model with independence

\[ p \quad \text{(independence)} \]
\[ f \quad (1 - f) \]

\[ (1 - p) \quad \text{(conformity)} \]

Lack of unanimity
\[ \epsilon \quad (1 - \epsilon) \]
Q-voter model with anticonformity

\[ p \rightarrow \text{anticonformity} \]

\[ (1 - p) \rightarrow \text{conformity} \]

\[ p \rightarrow \text{Lack of unanimity} \]

\[ (1 - p) \rightarrow \text{Lack of unanimity} \]
The mean-field approach

\[ c(t) = \frac{N_+(t)}{N}, \quad m(t) = \frac{1}{N} \sum_{i=1}^{N} S_i(t) = 1 - c(t) \]

\[ \gamma^+ = Prob \left( c \rightarrow c + \frac{1}{N} \right) \]
\[ \gamma^- = Prob \left( c \rightarrow c - \frac{1}{N} \right) \]
\[ \gamma^0 = Prob(c \rightarrow c) = 1 - \gamma^+ - \gamma^- \]
The mean-field approach for $N \to \infty$

**Model A: anticonformity**

$\gamma^+ = (1 - c)[(1 - p)c^q + p(1 - c)^q]$  
$\gamma^- = c[(1 - p)(1 - c)^q + pc^q]$  

**Model B: independence**

$\gamma^+ = (1 - c)[(1 - p)c^q + pf]$  
$\gamma^- = c[(1 - p)(1 - c)^q + pf]$  

**Time evolution**

$c \left( t + \frac{1}{N} \right) = c(t) + \frac{1}{N}(\gamma^+ - \gamma^-)$
Type of non-conformity matters
Nyczka et al. (2012, PRE)
Conformity and non-conformity

Model 1 continuous phase transition for any $q$

Model 2 continuous phase transition for any $q \leq 5$ and discontinuous phase transition for $q > 5$


(c) P. Nyczka, 2015
Is the voter model a model for voters?

- Weiss et al., PRX 4, 041008 (2014)
- Fernández-Gracia, PRL 112, 158701 (2014)
- U.S. presidential elections from 1980 to 2012
- Model calibration – noise level $D=0.03$
- What is the level of noise in the US?
- Maybe try with the $q$-voter model?
- What is the best value of $q$?
Question

• Debate important for psychologists
• Is it important from the macroscopic (societal) point of view?
• Do the modeling assumptions on social interactions (personal traits vs. situation) have impact on the system as a whole?
• Answer within the q-voter model
Person-situation debate

Two types of spinsons:
• permanently independent
• always susceptible to group pressure

Homogenous spinsons:
• The level of independence is the same for every spinson
Person vs. situation

Heterogeneous spinsons:
\[ \langle p \rangle = 0.2 \]

Homogeneous spinsons:
\[ \langle p \rangle = 0.2 \]

K. Sznajd-Weron, J. Szwabiński, R. Weron (2014) PLoS ONE 9(11), e112203
Person approach – independent

\[ S_i(t + 1) = -S_i(t) \text{ with probability } f \]
\[ S_i(t + 1) = S_i(t) \text{ with probability } 1 - f \]
Person approach— conformist

Follow unanimous group
Independence with probability $p$:
- $S_i(t + 1) = -S_i(t)$ with probability $f$
- $S_i(t + 1) = S_i(t)$ with probability $1 - f$

Conformity with probability $1-p$: follow unanimous group
The mean field approach

\[ \frac{N^\uparrow(t)}{N} = c(t) \]

\[ c(t + 1) = c(t) + \frac{1}{N} pf(1 - 2c(t)) \]

\[ + \frac{1}{N} (1 - p) \left[ c^q(t)(1 - c(t)) - (1 - c(t))^q c(t) \right] \]
Two groups

\[ N_1 + N_2 = N \]

Independent

\[ N_1 = pN \]

\[ N_1\uparrow(t) + N_1\uparrow(t) = pN \]

\[
N_1\uparrow(t + 1) = N_1\uparrow(t) + \frac{N_1\uparrow(t)}{N} f - \frac{N_1\uparrow(t)}{N} f
\]

\[
N_2\uparrow(t + 1) = N_2\uparrow(t) + \frac{N_2\uparrow(t)}{N} c^q(t) - \frac{N_2\uparrow(t)}{N} \left(1 - c(t) \right)^q
\]

\[
\frac{N_1\uparrow(t) + N_2\uparrow(t)}{N} = c(t)
\]
Person or situation?

\[
\frac{N^\uparrow(t)}{N} = c(t), \quad c(0) = 1, \quad c(\infty) \equiv c
\]
Thank you for your attention

It is over - simplified!

Why?

Keep it short and simple!