Uncertainty of the Sagnac interferometer-based measurement of the modal birefringence of polarization maintaining optical fibers

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Presented is an analysis of the expanded uncertainty of an indirect measurement of modal birefringence of polarization maintaining fibers, based on the Sagnac interferometer and an optical spectrum analyzer. By using this method it is easy to achieve an expanded measurement uncertainty considerably lower than 1%, at a confidence level of 0.95, in measurements of fibers, whose birefringence is independent of the wavelength, as well as fibers, whose birefringence depends on the wavelength, including birefringent photonic crystal fibers (PCF). Guidelines are given, on how to minimize the measurement uncertainty of modal birefringence of these groups of birefringent fibers.

Keywords: polarization maintaining fiber, photonic crystal fiber (PCF), modal birefringence, Sagnac interferometer, measurement uncertainty.

1. Introduction

Modal birefringence is an important parameter of an optical fiber. Its reliable and accurate measurement has a great significance in fiber optic communications as well as fiber optic sensors. A phase modal birefringence and a group modal birefringence is distinguished. The phase modal birefringence $\Delta n$ of an optical fiber is defined as the difference between effective indices of refraction $n_x$ and $n_y$ of the orthogonal polarization modes $\Delta n = n_x - n_y = (\beta_x - \beta_y)/k_0$, where $\beta_x$ and $\beta_y$ are the propagation constants of the polarization modes, $k_0 = 2\pi/\lambda$ and $\lambda_0$ is the wavelength in free space. The group modal birefringence is defined as $\Delta n_g = [\Delta n - \lambda_0(d\Delta n/d\lambda)]$. Another parameter characterizing a fiber is the beat length, which is the length of the fiber, after which the input state of polarization is reproduced. The beat length $L_B = 2\pi/(\beta_x - \beta_y)$ is directly related to the modal birefringence by the expression $L_B = \lambda_0/\Delta n$. The birefringent properties of a fiber can be determined by measuring the modal birefringence or by measuring the beat length. A number of methods were developed to measure those quantities (the beat length and modal birefringence of fibers). Among the methods of measuring the beat length, the ones often employed are the method of direct
observation of the Rayleigh backscatter image [1], the elasto-optic method [2] and its modifications [3, 4] and the wavelength scanning method [5]. In measurements of modal birefringence interferometric methods are utilized [6–8]. It needs to be noted that group modal birefringence and phase modal birefringence of birefringent fibers can be positive or negative, which depends on the type of the fiber and the wavelength range, in which it is operating [9–12]. The high number of those methods of measuring modal birefringence and beat length results from the large variety of fibers currently used and from the wide measuring range, covering 5 orders of magnitude of the measured birefringence. Most of those methods require a complex measuring arrangement and a careful adjustment of the measuring system. The method of measuring modal birefringence based on the Sagnac interferometer features a simple measuring system and easy operation. It allows the measurement of the absolute value of the group modal birefringence of optical fibers. For fibers, whose birefringence does not depend on the wavelength in the assumed wavelength range, that is the stress-induced birefringent fibers, the method enables the measurement of the phase modal birefringence. As is shown in the paper, it is also a method allowing the implementation of measurements of modal birefringence with low uncertainty of less than 1% at a confidence level of 0.95.

2. Theoretical fundamentals and the measuring system

The fiber optic Sagnac loop with a birefringent fiber operates like a multi-wavelength (multi-frequency) pass-block filter. This follows from the dependence of the phase difference between the polarization modes propagating in a birefringent fiber on the wavelength. The input light beam is divided in the 3 dB coupler into two beams propagating in opposite directions, which recombine in the coupler after traveling through the loop, including the birefringent fiber, creating an interference pattern dependent on the phase difference between the polarization modes. The transmission coefficient \( T(\lambda) \) of the Sagnac loop with a polarization maintaining fiber is a periodic function of the wavelength, given by the formula [13]

\[
T(\lambda) = \frac{1 - \cos(\delta)}{2}
\]

where \( \delta \) is the phase difference between the polarization components, propagating in a polarization maintaining fiber of length \( L \), defined by the relation \( \delta = \frac{2\pi}{\lambda}\Delta n L \).

It should be emphasized that the oscillations in the spectrum are related to the group birefringence of the polarization maintaining fiber, which can be demonstrated by calculating the derivative of the phase difference with respect to the wavelength

\[
\frac{d\delta}{d\lambda} = \frac{2\pi L}{\lambda^2} \left[ \frac{d\Delta n}{d\lambda} - \Delta n \right] = \frac{2\pi L}{\lambda^2} (-\Delta n_g)
\]
A single period of the spectrum oscillations \( \Lambda \) corresponds to a change in the phase of \( \Delta \delta = 2\pi \), therefore on the basis of Eq. (2) the absolute value of the group birefringence can be written in the form of the relation

\[
|\Delta n_g| = \frac{\lambda^2}{AL}
\]  

(3)

The measuring system for determining the modal birefringence of optical fibers based on a conventional Sagnac loop is introduced in Fig. 1.

![Fig. 1. Experimental setup for the Sagnac interferometer-based modal birefringence measurement method.](image1)

The Sagnac loop is made from a 3 dB fiber optic coupler, a polarization controller and a section of the fiber, which is to be measured. A beam of light from a wideband source is fed to one of the coupler inputs. The spectrum analyzer connected to the other coupler input enables a measurement of the spectrum period \( \Lambda \) of the loop’s transmitted beam and the wavelength \( \lambda \). Figure 2, in which a segment of the transmitted spectrum

![Fig. 2. Measured segment of the transmission spectrum of the Sagnac interferometer with a birefringent fiber.](image2)
of a Sagnac loop with a 35.0 cm long segment of a photonic crystal fiber (PCF) is shown, illustrates the measurement.

The period of the spectrum is determined from the relation

$$\Lambda = \lambda_2 - \lambda_1$$

(4)

The selection of the value of the period in birefringence measurements of fibers, whose birefringence depends on the wavelength, is influenced by two conflicting requirements: low uncertainty of the period measurement, and a good estimation of the correct value of birefringence for the wavelength $\lambda$, using the measured birefringence value, averaged over the period. A low uncertainty of the spectrum period measurement is achieved at a sufficiently high value of that period, while a good estimate of the correct value of birefringence for the wavelength $\lambda$ is achieved for a suitably small period of the spectrum. In birefringence measurements of fibers, whose birefringence does not depend on the wavelength, the choice of the period value is determined only by the requirement regarding the uncertainty of the measurement of that period. The period of the spectrum is altered by changing the length of the birefringent fiber to be measured.

The measurement of the segment of the fiber to be examined is usually performed by means of a linear ruler or a measuring tape. For practical reasons the investigated fiber is usually arranged in a loop and connected to a coupler and a polarization controller. In such a case, the induced birefringence of the fiber, caused by its bending, would need to be taken into account in the measurement. The birefringence calculated from relation (3) is the sum of the intrinsic birefringence and the induced birefringence caused by bending of the fiber. The birefringence of a silica fiber with an outer diameter of $2r$, being bent into a circle of radius $R$, can be calculated from the relation [6]

$$\Delta n_{\text{bent}} = 0.093 \left( \frac{r}{R} \right)^2$$

(5)

For fibers with an outer diameter of 250 $\mu$m wound into a coil with a radius of 8 cm, the birefringence induced by bending is $\Delta n_{\text{bent}} = 9.0 \times 10^{-7}$. In measurements of Bow–Tie and Panda type birefringent fibers and birefringent PCFs, whose birefringence values are within the range $(3-9) \times 10^{-4}$, the effect of their bending into a loop with a diameter greater than 8 cm can be neglected.

3. Expanded uncertainty of birefringence measurement

Calculating the modal birefringence value, on the basis of relation (3), is carried out by performing multiple measurements $\lambda$, $\lambda_1$ and $\lambda_2$ and then calculating their arithmetic mean values. On the basis of relation (4) the arithmetic mean value of the period of the spectrum is calculated. The calculated arithmetic mean values $\Lambda = \overline{\Lambda}$, $\lambda = \overline{\lambda}$ are inserted into relation (3). It was assumed that the values $\Lambda$ and $\lambda$ are not correlated, even though they are measured using the same instrument, because of the sequential measurement of those quantities. The measurement of the length $L$ of
the fiber segment is performed once. The expanded uncertainty of the modal birefringence measurement was estimated using the approximate method of estimating indirect measurements given in [14], because of the significantly lower man-hours than the method of calculating the measurement uncertainty according to the guide [15], while maintaining sufficient agreement between the calculation results of both methods. In the approximate methods, the coverage factor for type $A$ uncertainty and the coverage factor for type $B$ uncertainty are calculated separately. The expanded type $A$ uncertainty for every directly measured mean value of the input quantity $\bar{x}_j$ is calculated from the expression

$$U_A(\bar{x}_j) = k_{A_j} u_A(\bar{x}_j)$$

in which $k_{A_j} = t_{p, v}$ is the coverage factor equal to the quantile $t_{p, v}$ of the Student’s $t$-distribution, whose value depends on the confidence level $p$ and the number of the degrees of freedom $v = (n-1)$ ($n$ is the number of measurement readings); $u(\bar{x}_j) = s_A(\bar{x}_j)$ is the standard type $A$ uncertainty of the mean value of the input quantity $\bar{x}_j$, equal to the standard deviation of that value.

Similarly, the expanded type $B$ uncertainty is expressed by the relation:

$$U_B(\bar{x}_j) = k_{B_j} u_B(\bar{x}_j)$$

in which $k_{B_j}$ is the coverage factor for the type $B$ uncertainty, $u_B(\bar{x}_j)$ is the standard type $B$ uncertainty of the mean value of the input quantity $\bar{x}_j$.

The expanded uncertainty of the measurement of the quantity $y = f(x_j)$ at a confidence level of $p = 0.95$, for the case, when type $A$ uncertainties are larger than type $B$ uncertainties, can be estimated on the basis of the relation [14]

$$U(y) = \sum_{j=1}^{m} c_j^2 \left[ U_A^2(\bar{x}_j) + U_B^2(\bar{x}_j) \right] = \sum_{j=1}^{m} c_j^2 \left[ U_A^2(\bar{x}_j) + (\Delta_{\text{max}}(\bar{x}_j))^2 \right]$$

where $c_j = \partial y / \partial x_j$ are the sensitivity coefficients, $\Delta_{\text{max}}(\bar{x}_j)$ are the maximum permissible errors with a rectangular distribution of the measuring instruments.

Since in measurements of modal birefringence based on the Sagnac interferometer, the type $A$ uncertainties are larger than type $B$ uncertainties, the expanded uncertainty at a confidence level of 0.95 can be estimated on the basis of relation (8), which, for birefringence, will take the form

$$U(B) = \sqrt{c_\lambda^2 U^2(\lambda) + c_A^2 U^2(A) + c_L^2 U^2(L)}$$

where the individual sensitivity coefficients are determined by the formulas:

$$c_\lambda = \frac{\partial B}{\partial \lambda} = \frac{2\lambda}{\Lambda L}, \quad c_A = \frac{\lambda^2}{\Lambda^2 L}, \quad c_L = \frac{\lambda^2}{\Lambda L^2}$$
The period of the spectrum is determined from relation (4), therefore the uncertainty of the measurement of the period of the spectrum is defined by the formula

$$U(A) = \sqrt{c_{\lambda_2}^2 U_2^2(\lambda_2) + c_{\lambda_1}^2 U_1^2(\lambda_1)}$$

(11)

where the individual coefficients of sensitivity are

$$c_{\lambda_1} = \frac{\partial A}{\partial \lambda_1} = -1, \quad c_{\lambda_2} = \frac{\partial A}{\partial \lambda_2} = 1$$

(12)

The wavelength \(\lambda\) is measured several times with a spectrum analyzer. On the basis of formula (6) the expanded type \(A\) uncertainty of the average value of the wavelength is

$$U_A(\bar{\lambda}) = k_{AA} u_A(\bar{\lambda})$$

(13)

where \(k_{AA} = t_{p,v}\) is the coverage factor for the type \(A\) uncertainty of the wavelength \(\lambda\) measurements, equal to the quantile of the Student’s \(t\)-distribution, and \(u_A(\bar{\lambda})\) is the standard type \(A\) uncertainty of the measurement of the average value of the wavelength.

The expanded uncertainty of the measurement of the wavelength is calculated from the relation

$$U(\bar{\lambda}) = \sqrt{U_A^2(\bar{\lambda}) + (\Delta_{max} \bar{\lambda})^2}$$

(14)

in \(\Delta_{max} \bar{\lambda}\) which is the maximum permissible error of the spectrum analyzer used to measure the wavelength \(\bar{\lambda}\). In the case of the analyzer used, \(\Delta_{max} \bar{\lambda} = 0.5\) nm.

The wavelengths \(\lambda_1\) and \(\lambda_2\) are, as before, measured several times with a spectrum analyzer. The expanded type \(A\) uncertainty of the average values of the wavelengths \(\bar{\lambda}_1\) and \(\bar{\lambda}_2\) is calculated from the formulas

$$U_A(\bar{\lambda}_1) = k_{A\lambda_1} u_A(\bar{\lambda}_1), \quad U_A(\bar{\lambda}_2) = k_{A\lambda_2} u_A(\bar{\lambda}_2)$$

(15)

in which \(k_{A\lambda_1}\), \(k_{A\lambda_2}\) are the coverage factors for the type \(A\) uncertainty of the measurements of the wavelengths \(\lambda_1\) and \(\lambda_2\), respectively, \(u_A(\bar{\lambda}_1)\), \(u_A(\bar{\lambda}_2)\) are the standard type \(A\) uncertainties of the average values of the wavelengths \(\bar{\lambda}_1\) and \(\bar{\lambda}_2\), respectively.

The expanded uncertainty of the measurement of the wavelength \(\bar{\lambda}_1\) and \(\bar{\lambda}_2\) is calculated from the relation

$$U(\bar{\lambda}_1) = \sqrt{U_A^2(\bar{\lambda}_1) + (\Delta_{max} \bar{\lambda}_1)^2}, \quad U(\bar{\lambda}_2) = \sqrt{U_A^2(\bar{\lambda}_2) + (\Delta_{max} \bar{\lambda}_2)^2}$$

(16)

where \(\Delta_{max} \bar{\lambda}_1\) and \(\Delta_{max} \bar{\lambda}_2\) are the maximum permissible errors of the analyzer used to measure the wavelengths \(\bar{\lambda}_1\) and \(\bar{\lambda}_2\), respectively. These errors are composed of the nonlinearity error and the resolution error of the analyzer. The nonlinearity error
of the applied analyzer equals 0.020 nm in the range of 40 nm. The measurements were taken in the wavelength range of 20 nm. Therefore the nonlinearity error, in the measured range, was assumed as 0.010 nm. As the resolution error, one half of the wavelength value distinguishable by the analyzer was taken, assuming that the A/D converter with a microprocessor, used in the analyzer, correctly performs the operation of rounding the measurement results. The resolution error equals 0.010 nm. Therefore the permissible errors, for the applied analyzer, are $\Delta_{\text{max}} \lambda_1 = \Delta_{\text{max}} \lambda_2 = 0.020 \text{ nm}$.

The measurements of the lengths of the segments of the investigated birefringent fibers were performed once using a measuring ruler with a maximum permissible error $\Delta_{\text{max}} L = 0.5 \text{ mm}$. The measurement uncertainty was affected only by the $B$-type uncertainty, which, at the confidence level of $p = 0.95$, is $U(L) = U_B(L) \equiv \Delta_{\text{max}} L = 0.5 \text{ nm}$.

**Table 1.** Results of measurements of the wavelength $\lambda$ and calculations of its expanded uncertainty.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\lambda_j [\text{m}] \times 10^{-9}$</th>
<th>$\bar{\lambda} [\text{m}] \times 10^{-9}$</th>
<th>$u_A(\bar{\lambda}) [\text{m}] \times 10^{-9}$</th>
<th>$k_{A\lambda}$</th>
<th>$U_A(\bar{\lambda}) [\text{m}] \times 10^{-9}$</th>
<th>$\Delta_{\text{max}}(\bar{\lambda}) [\text{m}] \times 10^{-9}$</th>
<th>$U(\bar{\lambda}) [\text{m}] \times 10^{-9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1549.920</td>
<td>1549.960</td>
<td>0.018</td>
<td>2.78</td>
<td>0.050</td>
<td>0.5</td>
<td>0.502</td>
</tr>
<tr>
<td>2</td>
<td>1549.920</td>
<td>1549.960</td>
<td>0.018</td>
<td>2.78</td>
<td>0.050</td>
<td>0.5</td>
<td>0.502</td>
</tr>
<tr>
<td>3</td>
<td>1549.960</td>
<td>1549.960</td>
<td>0.018</td>
<td>2.78</td>
<td>0.050</td>
<td>0.5</td>
<td>0.502</td>
</tr>
<tr>
<td>4</td>
<td>1550.00</td>
<td>1549.960</td>
<td>0.018</td>
<td>2.78</td>
<td>0.050</td>
<td>0.5</td>
<td>0.502</td>
</tr>
<tr>
<td>5</td>
<td>1550.00</td>
<td>1549.960</td>
<td>0.018</td>
<td>2.78</td>
<td>0.050</td>
<td>0.5</td>
<td>0.502</td>
</tr>
</tbody>
</table>

**Table 2.** Results of measurements of the wavelength $\lambda_1$ and calculations of its expanded uncertainty.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\lambda_{1j} [\text{m}] \times 10^{-9}$</th>
<th>$\bar{\lambda}_1 [\text{m}] \times 10^{-9}$</th>
<th>$u_A(\bar{\lambda}_1) [\text{m}] \times 10^{-9}$</th>
<th>$k_{A\lambda}$</th>
<th>$U_A(\bar{\lambda}_1) [\text{m}] \times 10^{-9}$</th>
<th>$\Delta_{\text{max}}(\bar{\lambda}_1) [\text{m}] \times 10^{-9}$</th>
<th>$U(\bar{\lambda}_1) [\text{m}] \times 10^{-9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1546.160</td>
<td>1546.152</td>
<td>0.015</td>
<td>2.78</td>
<td>0.042</td>
<td>0.020</td>
<td>0.047</td>
</tr>
<tr>
<td>2</td>
<td>1546.120</td>
<td>1546.120</td>
<td>0.015</td>
<td>2.78</td>
<td>0.042</td>
<td>0.020</td>
<td>0.047</td>
</tr>
<tr>
<td>3</td>
<td>1546.120</td>
<td>1546.152</td>
<td>0.015</td>
<td>2.78</td>
<td>0.042</td>
<td>0.020</td>
<td>0.047</td>
</tr>
<tr>
<td>4</td>
<td>1546.200</td>
<td>1546.152</td>
<td>0.015</td>
<td>2.78</td>
<td>0.042</td>
<td>0.020</td>
<td>0.047</td>
</tr>
<tr>
<td>5</td>
<td>1546.160</td>
<td>1546.152</td>
<td>0.015</td>
<td>2.78</td>
<td>0.042</td>
<td>0.020</td>
<td>0.047</td>
</tr>
</tbody>
</table>

**Table 3.** Results of measurements of the wavelength $\lambda_2$ and calculations of its expanded uncertainty.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\lambda_{2j} [\text{m}] \times 10^{-9}$</th>
<th>$\bar{\lambda}_2 [\text{m}] \times 10^{-9}$</th>
<th>$u_A(\bar{\lambda}_2) [\text{m}] \times 10^{-9}$</th>
<th>$k_{A\lambda}$</th>
<th>$U_A(\bar{\lambda}_2) [\text{m}] \times 10^{-9}$</th>
<th>$\Delta_{\text{max}}(\bar{\lambda}_2) [\text{m}] \times 10^{-9}$</th>
<th>$U(\bar{\lambda}_2) [\text{m}] \times 10^{-9}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1554.080</td>
<td>1554.080</td>
<td>0.013</td>
<td>2.78</td>
<td>0.036</td>
<td>0.020</td>
<td>0.041</td>
</tr>
<tr>
<td>2</td>
<td>1554.080</td>
<td>1554.080</td>
<td>0.013</td>
<td>2.78</td>
<td>0.036</td>
<td>0.020</td>
<td>0.041</td>
</tr>
<tr>
<td>3</td>
<td>1554.120</td>
<td>1554.080</td>
<td>0.013</td>
<td>2.78</td>
<td>0.036</td>
<td>0.020</td>
<td>0.041</td>
</tr>
<tr>
<td>4</td>
<td>1554.040</td>
<td>1554.080</td>
<td>0.013</td>
<td>2.78</td>
<td>0.036</td>
<td>0.020</td>
<td>0.041</td>
</tr>
<tr>
<td>5</td>
<td>1554.080</td>
<td>1554.080</td>
<td>0.013</td>
<td>2.78</td>
<td>0.036</td>
<td>0.020</td>
<td>0.041</td>
</tr>
</tbody>
</table>
The budget of the expanded uncertainty, at a confidence level of 0.95, of the measurement of the modal birefringence of the PCF PM-1550-01 for the wavelength 1550 nm is presented in Tables 1–5.

### 4. Examples of measurements

Example measurements of modal birefringence based on the Sagnac interferometer together with the analysis of the uncertainty were performed for two types of birefringent fibers: a PCF PM-1550-01 and a conventional Panda-type fiber PM1550-HP (Nufern). The measurements were performed for three wavelengths 1.55 μm, 1.52 μm and 1.49 μm. The results of these measurements and the results of calculations of their uncertainty are given in Tables 6 and 7.

From the performed measurements and calculated uncertainties it follows that the method of measuring birefringence based on the Sagnac interferometer with the use of an optical spectrum analyzer allows to determine the modal birefringence of birefringent fibers with an expanded relative uncertainty of less than 1%, at a confidence level of \( p = 0.95 \). The main component of the uncertainty of the birefringence measurement when using this method is the uncertainty of the measurement of the period of the spectrum, which is determined from the measurement of the difference of two quantities (wavelengths) measured directly, whose values are not far from each other. Therefore the length of the investigated fiber should be chosen such, that the period of the spectrum is as large as possible, while paying attention to the fact

<table>
<thead>
<tr>
<th>Measured quantity ( X_j )</th>
<th>Average value ( \bar{x}_j )</th>
<th>Sensitivity coefficient ( c_j )</th>
<th>Expanded uncertainty ( U(\bar{x}_j) )</th>
<th>( c_j^2 U^2(\bar{x}_j) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength ( \lambda_1 )</td>
<td>1546.152×10⁻⁹ m</td>
<td>0.047×10⁻⁹ m</td>
<td>0.002209×10⁻¹⁸ m²</td>
<td></td>
</tr>
<tr>
<td>Wavelength ( \lambda_2 )</td>
<td>1554.080×10⁻⁹ m</td>
<td>0.041×10⁻⁹ m</td>
<td>0.001681×10⁻¹⁸ m²</td>
<td></td>
</tr>
<tr>
<td>Period of the spectrum ( \Lambda )</td>
<td>7.928×10⁻⁹ m</td>
<td>( \sum c_j^2 U^2(\bar{x}_j) = 0.003890×10⁻¹⁸ m² )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Uncertainty of the period of the spectrum \( \Lambda \): \( U(\Lambda) = 0.062×10⁻⁹ m \)

<table>
<thead>
<tr>
<th>Measured quantity ( X_j )</th>
<th>Average value ( \bar{x}_j )</th>
<th>Sensitivity coefficient ( c_j )</th>
<th>Expanded uncertainty ( U(\bar{x}_j) )</th>
<th>( c_j^2 U^2(\bar{x}_j) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength ( \lambda )</td>
<td>1549.960×10⁻⁹ m</td>
<td>1.117×10⁻⁵ m⁻¹</td>
<td>0.502×10⁻⁹ m</td>
<td>0.314×10⁻¹²</td>
</tr>
<tr>
<td>Period of the spectrum ( \Lambda )</td>
<td>7.928×10⁻⁹ m</td>
<td>1.092×10⁻⁵ m⁻¹</td>
<td>0.062×10⁻⁹ m</td>
<td>46.36×10⁻¹²</td>
</tr>
<tr>
<td>Length of the fiber ( L )</td>
<td>0.350 m</td>
<td>2.474×10⁻³ m⁻¹</td>
<td>0.5×10⁻³ m</td>
<td>1.53×10⁻¹²</td>
</tr>
<tr>
<td>Birefringence ( \Delta n_g )</td>
<td>8.657×10⁻⁴</td>
<td>( \sum c_j^2 U^2(\bar{x}_j) = 48.2×10⁻¹² )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Expanded uncertainty of modal birefringence \( U(\Delta n_g) = 6.9×10⁻⁶ \), \( U_e(\Delta n_g) = 0.8\% \)

The budget of the expanded uncertainty, at a confidence level of 0.95, of the measurement of the modal birefringence of the PCF PM-1550-01 for the wavelength 1550 nm is presented in Tables 1–5.
that increasing the range of the optical spectrum analyzer decreases its resolution and increases the component of the uncertainty stemming from the measurement of the length of the fiber. The optical spectrum analyzer applied for the measurement should feature a good spectral resolution and low nonlinearity. In measurements of birefringent fibers, whose birefringence depends on the wavelength, when minimizing the uncertainty of the birefringence measurement by increasing the period of the spectrum, one has to take into account the effect of averaging for the period of the spectrum. From the calculations of the uncertainty of the birefringence measurement it follows that with increasing wavelength, the uncertainty of the birefringence measurement decreases for the PCF, and increases for the Panda-type fiber, at the same uncertainty of the period measurement. The obtained result of the measurement of the absolute value of the group modal birefringence of the PM-1550-01 PCF at a wavelength of 1550 nm, \( \Delta n_g = (8.66 \pm 0.69) \times 10^{-4} \), is consistent with the results of measurements of the birefringence of that particular fiber obtained with the wavelength scanning method and published in [16, 17]. The value of the birefringence given there is \( \Delta n_g = 8.65 \times 10^{-4} \).

The increase in the absolute value of the birefringence with increasing wavelength observed in the measurements of the PCF is consistent with the results of studies of birefringent PCFs published in [11, 18]. The results of the measurements of a Panda-type conventional birefringent fiber confirm that the modal birefringence of such fibers does not depend on the wavelength.
5. Conclusions

The results of the performed measurements of the modal birefringence of polarization maintaining fibers and the results of the uncertainty calculations of these fibers indicate that the method of measuring birefringence based on the Sagnac interferometer and the use of an optical spectrum analyzer enables to determine the modal birefringence of birefringent fibers with the expanded relative uncertainty smaller than 1% at a confidence level of $p = 0.95$. The above results also indicate that the method can be used in measurements of birefringent fibers whose birefringence is independent of the wavelength and fibers whose birefringence is dependent on the wavelength, including PCFs. Because the main contribution to the uncertainty of the birefringence measurement comes from the uncertainty of the measurement of the period of the spectrum of the Sagnac loop, its value should be minimized first of all. In measurements of birefringent fibers, whose birefringence depends on the wavelength, the minimization of the uncertainty of the birefringence measurement by increasing the period of the spectrum limits the effect of averaging for the period of the spectrum. Because of the simplicity of the configuration, the easy way to perform the measurement, and the relatively small measurement uncertainty, the method can be applied in scientific research as well as engineering.

References

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